Zero correlation zone sequences based on interleaved perfect polyphase sequences

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Abstract

Zero Correlation Zone (ZCZ) sequences have applications in Quasi-Synchronous Code-Division Multiple-Access (QS-CDMA) communication systems. Many ZCZ sequences by interleaving perfect sequences have been investigated. In this paper, we focus on perfect polyphase sequences. By utilizing the inner structure of a perfect polyphase sequence, we derive a new construction of ZCZ sequences based on interleaving technique. Optimal ZCZ sequence sets can be obtained with respect to the Tang-Fan-Matsufuji bound. Furthermore, the condition under which the ZCZ sequence set is optimal is looser, compared with the previous results, which implies that our construction can produce optimal ZCZ sequence sets with new parameters..

1 Introduction

Sequences with desirable auto-correlation and cross-correlation properties have applications in communications and radar systems for identification, synchronization, ranging, or interference mitigation. Sequences with Zero Correlation Zone (ZCZ) have zero auto-correlation and cross-correlation simultaneously in some smaller zone around the origin. In QS-CDMA communication systems, a time delay between the signals of different users within a few chips is allowed. ZCZ sequences can be employed in such systems for eliminating both multiple access interference and multipath interference. In telecommunication industry, ZCZ sequences have been used as uplink random access channel preambles in the fourth-generation cellular standard LTE [12].

Numerous ZCZ sequence families have been proposed. The methods based on complementary sequence sets were reported in [1, 2, 14, 13]. There were also constructions derived by manipulating perfect sequences [4, 5, 6, 7, 9, 10, 15, 17, 19]. Among these, interleaving technique was employed in [5, 15, 17, 19]. Tang and Mow [15] proposed a construction by interleaving a perfect sequence via a shift sequence. The generated set is optimal with respect to the Tang-Fan-Matsufuji bound [16], when the length of the perfect sequence is coprime to the length of the shift sequence. Later in [5, 19], the size of the sequence set was increased by using a set of shift sequences instead of a single shift sequence. In this paper, we achieve the same goal by interleaving a perfect polyphase sequence via a single shift sequence. By utilizing the inner structure of the perfect polyphase sequence, we obtain ZCZ sequences with a large size. Furthermore, the condition under which the ZCZ sequence set is optimal is that a divisor of the length of the perfect polyphase sequence is coprime to the length of the shift sequence, which is looser than that in [15]. Compared with the results in [5, 19], the condition ensuring the optimal property in this paper is simpler.

The rest of this paper is organised as follows. Section 2 introduces preliminaries that are required for the subsequent sections. Based on interleaving technique and perfect polyphase sequences, we propose a new approach to constructing ZCZ sequences in Section 3. Finally, Section 4 concludes this paper.

2 Preliminaries

Let $\mathbb{Z}_N$ denote the ring of integers modulo N, where N is a positive integer. Let $\mathbf{a} = \{a(t)\}_{t=0}^{N-1}$ and $\mathbf{b} = \{b(t)\}_{t=0}^{N-1}$ be two complex sequences of period N. The (periodic) *cross-correlation* (CC) of $\mathbf{a}$ and $\mathbf{b}$ at shift τ is defined as

$$R_{\mathbf{a},\mathbf{b}}(\tau) = \sum_{t=0}^{n-1} a(t+\tau)b^*(t), \qquad 0 \le \tau < N,$$

where $t+\tau$ is reduced modulo N and $b^*(t)$ is the complex conjugate of the complex number $b(t)$. When $\mathbf{a} = \mathbf{b}$, $R_{\mathbf{a}}(\tau) = R_{\mathbf{a},\mathbf{a}}(\tau)$ is called the *auto-correlation* (AC) of $\mathbf{a}$.

A sequence $\mathbf{a}$ is said to be *perfect* if $R_{\mathbf{a}}(\tau) = 0$ for all $0 < \tau < N$. A sequence set $\mathcal{S}$ is called *periodically uncorrelated* if the correlation between any two distinct sequences in $\mathcal{S}$ is equal to zero at any shift.

According to the Sawarte bound [11], it is impossible to have sequences which have simultaneously zero out-of-phase AC and zero CC during an entire period. In other words, there exists no uncorrelated sequence set in which each sequence is also perfect. However, there do exist sets of sequences satisfying both properties simultaneously in some smaller zone around the origin (called the zero-correlation zone, or ZCZ).

Definition 1. *Let $\mathcal{S} = \{\mathbf{s}_0, \mathbf{s}_1, \cdots, \mathbf{s}_{M-1}\}$ be a sequence set of size M of length N. The set $\mathcal{S}$ is called an (N, M, Z)-ZCZ sequence set if*

$$R_{\mathbf{s}_i}(\tau) = 0 \;\; \textit{for } (0 < |\tau| < z_a)$$

and

$$R_{\mathbf{s}_i,\mathbf{s}_j}(\tau) = 0 \;\; \textit{for } (0 \le |\tau| < z_c \textit{ and } i \ne j),$$

where $Z = \min(z_a, z_c)$ is called the length of the zero correlation zone.

Given an (N, M, Z)-ZCZ set, the Tang-Fan-Matsufuji bound implies that $MZ \leq N$ [16]. A ZCZ set meeting this bound is said to be *optimal*.

In the following, we introduce a construction of ZCZ sequences based on interleaved perfect sequences [15], after which a unified construction of perfect polyphase sequences is presented [8]. With some desirable properties of the perfect polyphase sequences, we propose a new interleaving approach and derive ZCZ sequences with new parameters in the next section.

2.1 ZCZ sequences based on interleaving technique

Since Gong proposed the concept of interleaved sequences in 1995 [3], interleaving technique has been an important tool for sequence design. A construction of ZCZ sequences based on interleaving technique by Tang and Mow is as follows [15].

Let $\mathbf{a} = (a_0, a_1, \cdots, a_{N-1})$ be a perfect sequence of length N. Let $\mathbf{e} = (e_0, e_1, \cdots, e_{M-1})$ be a sequence of length M defined over $\mathbb{Z}_N = \{0, 1, \cdots, N-1\}$. Let L be the left cyclic shift operator such that $L^{e_i}(\mathbf{a})$ denotes the e_i-element left cyclically shifted version of $\mathbf{a}$. Then we can obtain an $N \times M$ matrix

$$L^{\mathbf{e}}(\mathbf{a}) = \begin{bmatrix} a_{e_0} & a_{e_1} & \cdots & a_{e_{M-1}} \\ a_{e_0+1} & a_{e_1+1} & \cdots & a_{e_{M-1}+1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{e_0+N-1} & a_{e_1+N-1} & \cdots & a_{e_{M-1}+N-1} \end{bmatrix}, \tag{1}$$

denoted by $L^{\mathbf{e}}(\mathbf{a}) = [L^{e_0}(\mathbf{a}) \sim L^{e_1}(\mathbf{a}) \sim \cdots \sim L^{e_{M-1}}(\mathbf{a})]$ for convenience.

By concatenating the successive rows of the matrix $L^{\mathbf{e}}(\mathbf{a})$, one obtains an interleaved sequence $\overline{\mathbf{a}}$ of length NM. We mathematically denote $\overline{\mathbf{a}} = I(L^{\mathbf{e}}(\mathbf{a}))$, where I denotes the interleaving operator, and $\mathbf{a}$ and $\mathbf{e}$ are called the *component* and *shift* sequences of $\overline{\mathbf{a}}$, respectively.

We write τ $(0 \leq \tau < NM)$ in the form of $M\tau_1 + \tau_2$, where $0 \leq \tau_1 < N$ and $0 \leq \tau_2 < M$. The auto-correlation function of $\overline{\mathbf{a}}$ at shift τ is given by

$$R_{\overline{\mathbf{a}}}(\tau) = \sum_{i=0}^{M-\tau_2-1} R_{\mathbf{a}}(e_{i+\tau_2} - e_i + \tau_1) + \sum_{i=M-\tau_2}^{M-1} R_{\mathbf{a}}(e_{i+\tau_2-M} - e_i + \tau_1 + 1).$$

We can see that $R_{\overline{\mathbf{a}}}(\tau) = 0$ for $0 < \tau < Z$ if $\mathbf{e}$ satisfies the condition:

$$\begin{cases} e_{t+\tau_2} - e_t + \tau_1 \not\equiv 0 \pmod N & \text{if } 0 \le t < M - \tau_2; \\ e_{t+\tau_2-M} - e_t + \tau_1 + 1 \not\equiv 0 \pmod N & \text{if } M - \tau_2 \le t < M, \end{cases}$$

for all $0 < \tau < Z$ (i.e., $M\tau_1 + \tau_2 < Z$ with $\tau_1 \ge 0$ and $0 \le \tau_2 < M$). Therefore, the key of the interleaving technique is to construct shift sequences satisfying the condition above.

A construction of shift sequences proposed by Tang and Mow [15] is as follows. Let $\mathbf{e}$ be a shift sequence of length M over $\mathbb{Z}_N$. Let $d = \lfloor (N-1)/M \rfloor$, where $\lfloor x \rfloor$ denotes the largest integer no larger than x. We denote d' an integer such that $N = dM + d'$ and define

$$h = \begin{cases} d' & \gcd(N, M) = 1, \\ d' - 1 & \text{otherwise.} \end{cases}$$

Let $u = \gcd(h, M) = 1$ and $v = M/u$. We write $0 \le t < M$ in the form of $ht_1 + t_2$, where $0 \le t_1 < v$ and $0 \le t_2 < u$ and define

$$e_t = \begin{cases} 0 & (t = 0) \text{ or } (d = 0 \text{ and } t = 0 (\text{mod } h)), \\ N - dt_1 - (d + \frac{h}{M})vt_2 - \lfloor ht_1/M \rfloor & \text{otherwise.} \end{cases} \quad (2)$$

Lemma 1. *[15] Let* $\mathbf{a}$ *be a perfect sequence of length* N *and* $\mathbf{e}$ *be a shift sequence of length* M *over* $\mathbb{Z}_N$ *defined by* (2). *Then the interleaved sequence* $\overline{\mathbf{a}} = I(L^{\mathbf{e}}(\mathbf{a}))$ *is an* $(NM, 1, N-1)$*-ZCZ sequence. In particular,* $\overline{\mathbf{a}}$ *is an* $(NM, 1, N)$*-ZCZ sequence when* $\gcd(N, M) = 1$.

A ZCZ set based on the interleaved sequence $\overline{\mathbf{a}}$ can be obtained in the following way. Let H be an $M \times M$ orthogonal matrix and $\mathbf{h}_i$ for $0 \le i < M$ be the $(i+1)$-th row of H. Let $\mathbf{h}_i \odot L^{\mathbf{e}}(\mathbf{a}) = [h_{i,0} L^{e_0}(\mathbf{a}) \sim h_{i,1} L^{e_1}(\mathbf{a}) \sim \cdots \sim h_{i,M-1} L^{e_{M-1}}(\mathbf{a})]$. Thus, a set of M matrices $\{\mathbf{h}_i \odot L^{\mathbf{e}}(\mathbf{a})\}_{i=0}^{M-1}$ can be derived and is denoted by $H \odot L^{\mathbf{e}}(\mathbf{a})$. By concatenating the successive rows of each matrix, a set $\mathcal{S} = \{\mathbf{s}_i = I(\mathbf{h}_i \odot L^{\mathbf{e}}(\mathbf{a})), \quad 0 \le i < M\}$ can be obtained and has the following property.

Lemma 2. *[15] Let* $\mathbf{a}$ *be a perfect sequence of length* N *and* H *be an* $M \times M$ *orthogonal matrix. The shift sequence* $\mathbf{e}$ *of length* M *is defined by* (2). *Then the set* $\mathcal{S}$ *from* $H \odot L^{\mathbf{e}}(\mathbf{a})$ *is an* $(NM, M, N-1)$*-ZCZ sequence set. In particular,* $\mathcal{S}$ *is an optimal* (NM, M, N)*-ZCZ set when* $\gcd(N, M) = 1$.

2.2 Perfect polyphase sequences

As previously mentioned in the introduction, many constructions of ZCZ sequences are based on perfect sequences. However, we focus on perfect polyphase sequences in this paper. A *polyphase sequence* is a sequence whose elements are are all complex roots of unity of the form $exp(i2\pi x)$ where x is a rational number and $i = \sqrt{-1}$. Mow [8] classified all the known perfect polyphase sequences into four classes and introduced a unified construction that includes all the four classes as follows.

Let r and m be any two positive integers. For any $l \in \mathbb{Z}_m$ and $k \in \mathbb{Z}_{rm}$, we define a function

$$f_l(k) = mc(r)\alpha(l)k^2 + \beta(l)k + g(l), \tag{3}$$

where $c(r)$ is 1 when r is odd and $\frac{1}{2}$ otherwise, α is any function over $\mathbb{Z}_r$ with $gcd(\alpha(l), r) = 1$ for all $l \in \mathbb{Z}_m$, $\beta : \mathbb{Z}_m \to \mathbb{Z}_{rm}$ is any function such that $l \mapsto \beta(l) \pmod{m}$ is a permutation over $\mathbb{Z}_m$, and g is any function over the rational numbers. Then the unified construction of perfect polyphase sequences is as follows.

Lemma 3. *[8] Let* ω_{rm} *be a primitive* rm*-th root of unity. A polyphase sequence* $\mathbf{c}$ *of period* rm^2 *defined by*

$$c(km + l) = \omega_{rm}^{f_l(k)} = \omega_{rm}^{mc(r)\alpha(l)k^2+\beta(l)k+g(l)}, \quad l \in \mathbb{Z}_m, k \in \mathbb{Z}_{rm},$$

is perfect.

In particular, we can write the sequence $\mathbf{c}$ in the form of an $rm \times m$ matrix such that $\mathbf{c}$ can be reproduced by concatenating successive rows of the matrix. Then each column can be regarded as a new sequence of length rm. Thus, we derive a sequence set $\mathcal{C} = \{\mathbf{c}_l = \{\omega_{rm}^{f_l(k)}\}_{k=0}^{rm-1}, \;\; 0 \le l < m\}$.

Lemma 4. *[8, 18] The set $\mathcal{C}$ has the following properties:*

1) For each $0 \le l < m$, we have

$$|R_{\mathbf{c}_l}(\tau)| = \begin{cases} 0 & \tau \not\equiv 0 \bmod r; \\ rm & \textit{otherwise}. \end{cases}$$

2) For any $0 \le l_1 \ne l_2 < m$, $|R_{\mathbf{c}_{l_1}\mathbf{c}_{l_2}}(\tau)| = 0$ for any τ. In other words, $\mathcal{U}$ is periodically uncorrelated.

3 A new construction of ZCZ sequences

We introduced a construction of ZCZ sequences by interleaving a perfect sequence via a shift sequence in the previous section. In this section, we look into the internal structure of a perfect polyphase sequence and write it in the form of a matrix. By interleaving the matrix, we derive ZCZ sequences with new parameters.

Let $\mathbf{c} = (c_0, c_1, \cdots, c_{N-1})$ be a perfect polyphase sequence of period $N = rm^2$, where r and m are any two positive integers. We denote C an $rm \times m$ matrix such that the sequence $\mathbf{c}$ can be reproduced by concatenating successive rows of the matrix. For convenience, we denote $C = [\mathbf{c}_0 \sim \mathbf{c}_1 \sim \cdots \sim \mathbf{c}_{m-1}]$, where $\mathbf{c}_i$ is the $(i+1)$-th column of the matrix C.

Let $\mathbf{e} = (e_0, e_1, \cdots, e_{M-1})$ be a shift sequence of length M defined over $\mathbb{Z}_r$ as in (2). Let $L^{e_i}(C)$ denote an $rm \times m$ matrix where each column is the e_i-element left cyclically shifted version of the corresponding column of C, i.e., $L^{e_i}(C) = [L^{e_i}(\mathbf{c}_0) \sim L^{e_i}(\mathbf{c}_1) \sim \cdots \sim L^{e_i}(\mathbf{c}_{m-1})]$. It follows that a $rm \times mM$ matrix can be derived by $L^{\mathbf{e}}(C) = [L^{e_0}(C) \sim L^{e_1}(C) \sim \cdots \sim L^{e_{M-1}}(C)]$. Thus, one obtains an interleaved sequence $\overline{\mathbf{c}} = I(L^{\mathbf{e}}(C))$ of length NM by concatenating the successive rows of the matrix $L^{\mathbf{e}}(C)$.

From the construction above, each column of the matrix $L^{\mathbf{e}}(C)$ is either a sequence in $\mathcal{C} = \{\mathbf{c}_i,\ 0 \le i < m\}$ or its cyclic shift. We know that the set $\mathcal{C}$ is uncorrelated by Lemma 4, i.e., the cross-correlation between any two distinct sequences in $\mathcal{C}$ is zero at all shifts. Therefore, the auto-correlation function $R_{\overline{\mathbf{c}}}(\tau)$ of the interleaved sequence $\overline{\mathbf{c}}$ is zero for any $\tau \not\equiv 0 \pmod{m}$. We have the following theorem for more detailed information about the auto-correlation of $\overline{\mathbf{c}}$.

Theorem 1. *The sequence $\overline{\mathbf{c}} = I(L^{\mathbf{e}}(C))$ is an $(NM, 1, rm)$-ZCZ sequence when $\gcd(r, M) = 1$ and an $(NM, 1, (r-1)m)$-ZCZ sequence otherwise.*

Proof. Let $\tau = mM \cdot \tau_3 + \tau_4$, where $0 \le \tau_3 < rm$ and $0 \le \tau_4 < mM$. When $\tau_4 \not\equiv 0 \pmod{m}$, the auto-correlation function of $\overline{\mathbf{c}}$ is equal to zero as we previously explained.

When $\tau_4 \equiv 0 \pmod{m}$, let $\tau_4 = m\tau_4'$ and then $\tau = mM \cdot \tau_3 + m\tau_4'$, where $0 \le \tau_3 < rm$ and $0 \le \tau_4' < M$. Then the auto-correlation function of $\overline{\mathbf{c}}$ at shift τ is given by

$$R_{\overline{\mathbf{c}}}(\tau) = \sum_{j=0}^{m-1} \left(\sum_{i=0}^{M-\tau_4'-1} R_{\mathbf{c}_j}(e_{i+\tau_4'} - e_i + \tau_3) + \sum_{i=M-\tau_4'}^{M-1} R_{\mathbf{c}_j}(e_{i+\tau_4'-M} - e_i + \tau_3 + 1) \right).$$

For each $0 \le j < m$, $R_{\mathbf{c}_j}(\tau) = 0$ for $\tau \not\equiv 0 \pmod{r}$ by Lemma 4. The shift sequence $\mathbf{e}$ is defined over $\mathbb{Z}_r$ as in (2). With a similar proof as for Lemma 1, we can prove that the inner sum above is zero for $0 < M\tau_3 + \tau_4' < Z$ for each $0 \le j < 0$, where Z is r when $\gcd(r, M) = 1$ and 0 otherwise. As a result, $R_{\overline{\mathbf{c}}}(\tau) = 0$ for $0 < \tau < m(r-1)/mr$. □

In the following, we generate a sequence set based on the interleaved sequence $\overline{\mathbf{c}}$ above by using orthogonal matrix. Orthogonal matrix is frequently used in the constructions of ZCZ sequence sets to preserve the orthogonality among distinct sequences in a set. Let H be an $mM \times mM$ orthogonal matrix, where $\mathbf{h}_n$ for $0 \le n < mM$ is the $(n+1)$-th row of the matrix H. Let $L^{\mathbf{e}}(C)$ be the $rm \times mM$ matrix that induces the sequence $\overline{\mathbf{c}}$. Then a new $rm \times mM$ matrix can be derived by the operation $\mathbf{h}_n \odot L^{\mathbf{e}}(C)$. Thus, we get a set of mM matrices $\{\mathbf{h}_n \odot L^{\mathbf{e}}(C)\}_{n=0}^{mM-1}$, denoted by $\mathbb{H} \odot L^{\mathbf{e}}(C)$. By concatenating the successive rows of each matrix, a set of mM sequences of length NM is generated, i.e., $\mathcal{S} = \{\mathbf{s}_n = I(\mathbf{h}_n \odot L^{\mathbf{e}}(C)) | 0 \le n < mM\}$.

Theorem 2. *The set* $\mathcal{S}$ *is an* (NM, mM, rm)*-ZCZ sequence when* $\gcd(r, M) = 1$ *and an* $(NM, mM, (r-1)m)$*-ZCZ sequence otherwise, where* $N = rm^2$*.*

Proof. Let $\mathbf{s}_{n_1}$ and $\mathbf{s}_{n_2}$ be two sequences in set $\mathcal{S}$. When $\tau = 0$, we have $R_{\mathbf{s}_{n_1}\mathbf{s}_{n_2}}(\tau) = rm \sum_{i=0}^{mM-1} h_{n_1,i}h_{n_2,i}$, which is zero when $n_1 \neq n_2$. Similar to the proof in Theorem 1, $R_{\mathbf{s}_{n_1}\mathbf{s}_{n_2}}(\tau) = 0$ when $\tau \not\equiv 0 \pmod{m}$.

When $\tau \equiv 0 \pmod{m}$ and $\tau \neq 0$, let $\tau = mM \cdot \tau_3 + m\tau_4'$, where $0 \leq \tau_3 < rm$ and $0 \leq \tau_4' < M$. The correlation function between $\mathbf{s}_{n_1}$ and $\mathbf{s}_{n_2}$ is given by

$$R_{\mathbf{s}_{n_1}\mathbf{s}_{n_2}}(\tau) = \sum_{j=0}^{m-1} \left(\sum_{i=0}^{M-\tau_4'-1} h_{n_1,im+j} h_{n_2,(i+\tau_4')m+j} R_{\mathbf{c}_j}(e_{i+\tau_4'} - e_i + \tau_3) + \sum_{i=M-\tau_4'}^{M-1} h_{n_1,im+j} h_{n_2,(i+\tau_4')m+j} R_{\mathbf{c}_j}(e_{i+\tau_4'-M} - e_i + \tau_3 + 1) \right).$$

Similar to the proof as in Theorem 1, we have $R_{\mathbf{s}_{n_1}\mathbf{s}_{n_2}}(\tau) = 0$ for $0 < \tau < m(r-1)/mr$. □

With a similar interleaving technique as in [15], we derive ZCZ sequences with new parameters by utilizing the inner structures of perfect polyphase sequences. The size of a ZCZ sequence set is increased when $m \neq 1$, compared with [15]. The constructed ZCZ sequence set has flexible choices on the size and the ZCZ length, which depends on the decomposition of the period N. Table 1 gives a comparison on parameters of some known constructions of optimal ZCZ sets based on perfect sequences.

Remark 1. *The condition under which the ZCZ sequence set is optimal with respect to the Tang-Fan-Matsufuji bound is* $\gcd(N, M) = 1$ *in [6, 9, 15]. In this paper, the condition is* $\gcd(r, M) = 1$ *with* $r = N/m^2$ *for a positive integer* m*, which is obviously looser. When* $\gcd(N, M) \neq 1$ *and* $\gcd(r, M) = 1$*, Theorem 2 yields new optimal ZCZ sequence sets.*

Remark 2. *From Table 1, we have the following observations:*

Table 1: Optimal ZCZ sequence sets based on perfect sequences

References	Period of ZCZ sequences	perfect sequence length	Set size	ZCZ length	Conditions on optimal sets
[9]	NM	$N = Q^2$	M	N	$\gcd(N, M) = 1$
[15]	NM	N	M	N	$\gcd(N, M) = 1$
[6]	NM	N	M	N	$\gcd(N, M) = 1$
[5, 19]	NM'	N	PM'	L	†
this paper	NM	$N = rm^2$	mM	$rm/rm - m$	$\gcd(r, M) = 1$

N, Q, r and m are positive integers;
† The parameters M', $P$and conditions in [5, 19] are given as below:
(1) When $M'|L$, $P = \left\lfloor \frac{N-M'-\sigma}{L} \right\rfloor$ and the condition for optimal ZCZ sequence sets is $L > M'(M' + \sigma + r)$, where $N = PL + M' + \sigma + r$ with $0 \leq r < L$, and σ is 0 when $M' = 2$ or $L|N - 1$ and 1 otherwise;
(2) When M' even and $L \equiv M'/2(\text{mod} M')$, $P = \left\lfloor \frac{N-3M'/2+1-\sigma'}{L} \right\rfloor$ and the condition for optimal ZCZ sequence sets is $L > M'(3M'/2 - 1 + \sigma' + r)$, where $N = PL + 3M'/2 - 1 + \sigma' + r$ with $0 \leq r < L$, and σ' is 0 when $M' = 2$ or 4 and 1 otherwise.

1) *When $m = 1$ and $N = r$, the parameters in Theorem 2 are the same as those in [6, 15]. When $m = 1$ and $N = r = Q^2$, the parameters in Theorem 2 are the same as those in [9]. When $m \neq 1$, the size of the ZCZ sequence set in this paper is larger than that in [6, 9, 15].*

2) *The size of a ZCZ sequence set was also increased in [5, 19] by using a set of shift sequences instead of a singe shift sequence. However, the condition on optimal sets in this paper is much simpler and thus can be easily verified. Moreover, ZCZ sequences in [5, 19] were obtained for two cases: a) $M'|L$ or b) M' even and $L \equiv M'/2(\text{mod} M')$, while in this paper M can be any positive integer. Therefore, we have more choices on the length of the constructed ZCZ sequences when a perfect sequence of length N is given.*

4 Conclusion

We presented a generic construction of ZCZ sequences based on interleaving technique and perfect polyphase sequences. With a similar technique employed in [15], we generated new ZCZ sequences with a larger size. The generic construction can generate optimal ZCZ sequences under a much looser condition, compared with some previous results. Thus, optimal ZCZ sequence sets with new parameters are derived as a result. Moreover, the constructed ZCZ sequences have flexible parameters with the size and the ZCZ length. Given a period of ZCZ sequences, we have flexible choices on the size and the ZCZ length for different applications.

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